Questions from the Written Competition for the Spring 2001 Pee-Dee Regional High-School Mathematics Tournament

Sponsored by
The Pee Dee Education Center
and
The Department of Mathematics at Francis Marion University

Students had one hour to solve these problems. Space was permitted on the written competition for students to work their problems. Only the questions are written here. Diagrams also appeared on the competition that the students had; these diagrams do not show up on this document.

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1. The larger circle with center O has an area of 420 square inches. The smaller circle contains the point O and is tangent to the larger circle. What is the area of the shaded part? [*Shaded area was probably that inside the larger circle but outside the smaller circle.* — Editor]

2. A person enters a skyscraper on the first floor. He climbs the stares to the 85th floor. How many flights of stairs did he use?

3. If x is a positive integer and its base ten logarithm is 5.3327, how many digits does x^{1000} have?

4. Consider the following product

$$239772843826156410124 \times 12454878754553 \,.$$

What is the tens digit of this product?

5. There are three books on a shelf. Each book has 480 pages. The first page of each book is 1 and the last page of each is 480. Mr. Bookworm is between pages 320 and 321 in Book 1. Ms. Bookworm is in Book 3 between pages 88 and 89. Mr. Bookworm chews a tunnel until he gets to Ms. Bookworm. How many pieces of paper does he chew through? (Assume that the book covers ***are not*** made of paper.)

6. In 1978, two students at California State University proved that $2^{11213} - 1$ is a prime number. What is the last digit of this number?

7. The following is the equation of a circle:

$$x^2 - 10x + y^2 + 6x - 2 = 0\,.$$

Where is the center and what is the radius of this circle?

8. A cylindrical drink can has a height which is twice its diameter. It has a volume equal to $216 \cdot \pi$. What are the diameter and height of the can?

9. Assume that $\log_k 1024 = 5$. What is the value of k?

10. Write the following repeating decimal as a fraction in lowest terms:

$$1.2\overline{37} = 1.23737373737\ldots$$

11. A supermarket supply house is running a special. It will sell a 12 ounce soft drink for 45 cents each. If you buy 10,000 cans, what will be your cost (in dollars)?

12. Suppose that $P(x) = x^4 - 7x^3 + 15x^2 - 18x - 35$. If $(x - 5)$ is a factor of $P(x)$ and $P(x) = (x - 5) \cdot Q(x)$, what is $Q(x)$?

13. In the diagram to the right the radius of the circle is 17. The line l_1 has the equation $y = -8$. What id the area of the triangle $\triangle ABC$? [*Diagram is missing; problem cannot be reconstructed.* — Editor]

14. See diagram to the right. Consider only the smallest individual cubes and assume solid stacks (no gaps). Determine the number of cubes **not visible** from the perspective shown. [*Diagram is missing; problem cannot be reconstructed.* — Editor]

15. People may move through the diagram along the direction of the arrows shown. Of the people that arrive at the point A $\frac{3}{5}$ take arrow I and $\frac{2}{5}$ take arrow 2. Of the people that arrive at the point B $\frac{1}{4}$ take arrow 3 and $\frac{3}{4}$ take arrow 4. Finally, those that arrive at point C $\frac{2}{3}$ take arrow 5 and $\frac{1}{3}$ take arrow 6. [*Neither the diagram nor the question are present here; the original question cannot be reconstructed.* — Editor]

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