Questions from the Written Competition for the Fall 2005 Pee-Dee Regional High-School Mathematics Tournament

Sponsored by

The Pee Dee Education Center

and

The Department of Mathematics at Francis Marion University

Students had one hour to solve these problems. Layout of what follows matches what students actually saw. Unfortunately, the notation "dot – dot – dot" renders as a capital lambda in this document, but it rendered correctly on the materials the students had in front of them on the day of the competition. — Editor.

1. In the following long division, what will be the remainder?

$11111\overline{)111111111111}$

Answer to Problem 1:

2. Rationalize the denominator:

$$\frac{1}{\sqrt{7}+\sqrt{5}}$$

Answer to Problem 2:

3. The circle C has equation $\{x^2 + y^2 - 6x + 4y + 4 = 0\}$. What is the circumference of C?

Answer to Problem 3:

4. Let $f(x) = x^2$ and $g(x) = 3x$.
Find and simplify $f \circ g \circ g \circ f(x)$, which is also the same as $f(g(g(f(x))))$.

Answer to Problem 4:

5. Simplify completely:

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}}$$

Answer to Problem 5:

6. Evaluate the following sum:

$$\sum_{k=1}^{100} (-1)^k \cdot k$$

Answer to Problem 6:

7. John walked from his house to the store at a speed of 4 feet per second. On the return trip he walked at a speed of 3 feet per second. It took him 21 minutes to walk the complete circuit. How far is the store from his house?

Answer to Problem 7:

8. The base-10 decimal expansion of the number one-third is $0.333333333\Lambda = 0.\overline{3}$.
What is the base-2 binary expansion of the number one-third?

Answer to Problem 8:

9. Simplify completely: $\log\left(\frac{1}{2}\right)+\log\left(\frac{2}{3}\right)+\log\left(\frac{3}{4}\right)+\ \Lambda\ +\log\left(\frac{98}{99}\right)+\log\left(\frac{99}{100}\right)$.
The base of the logarithm in each case is 10.

Answer to Problem 9:

10. Remember that $(n!) = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1$. The number (995!) ends in 246 zeros when written out base ten. How many zeros are at the end of the number (1005!) when *it* is written out base ten?

Answer to Problem 10:

11. Solve for x, where x is restricted to be a *positive* real number:

$\log_x (5 - 2x) = 2$

Answer to Problem 11:
$x =$

12. Simplify the following expression until it contains only a single radical:

$\sqrt{6 + \sqrt{11}} - \sqrt{6 - \sqrt{11}}$

Answer to Problem 12:

13. Solve for x and y simultaneously in the following system:

$$2x^3 + 3y = 23$$
$$3x^3 + 2y = 2$$

Answer to Problem 13:
$x =$ and $y =$

14. Grains of wheat are placed upon 4×4 chess board grains as follows: one grain is placed upon square 1, two grains upon square 2, four grains upon square 3, eight grains upon square 4, and so forth. How many more grains of wheat are on the white squares than on the black squares?

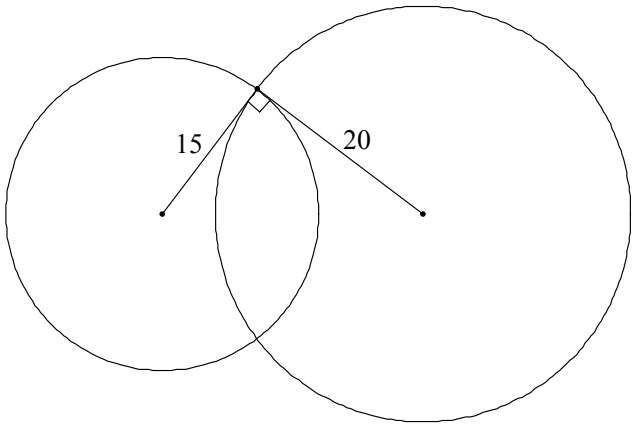
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Answer to Problem 14:

15. A Florida's driver's license uses the driver's gender, birth month, and birth day number to form the last three digits of the driver's license number. For a male the last three digits are calculated by the formula $40(m - 1) + d$. Here m is the month number (1 = January, 2 = February, etc) and d is the number of the day in that month. For example Howard's birthday is September 16 the last three digits would be computed as $40(9 - 1) + 16 = 336$. Determine the birth month and day of a male whose driver's licence number in Florida ends with the number 295.

Answer to Problem 15:
Month = Day =

16. A circle of radius 15 intersects another circle of radius 20 at right angles. What is the difference of the areas of the non-overlapping portions?



Answer to Problem 16: