

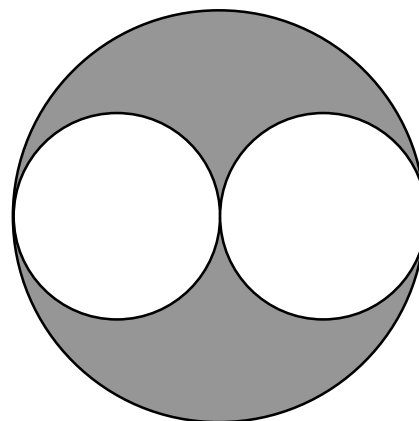
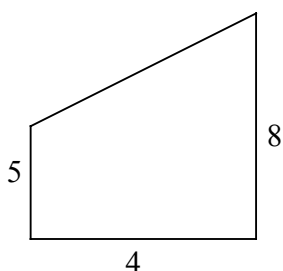
Questions from the Written Competition for the Fall 1999 Pee-Dee Regional High-School Mathematics Tournament

Sponsored by
The Pee Dee Education Center
and
The Department of Mathematics at Francis Marion University

Students had one hour to solve these problems. Space was permitted on the written competition for students to work their problems.



1. The trapezoid below has the lengths of three sides given. What is the perimeter of this trapezoid?



2. In the drawing on the right, the two small circles are tangent to each other at the center of the largest circle. The two small circles are tangent to the larger circle. The diameter of the small circles are 10 inches. The area A of the shaded region is $A = k\pi$. What is the value of k ?
3. Assume that θ is in the second quadrant and $\sin \theta = \frac{3}{5}$. What is the value of $\sin(2\theta)$?
4. Write the following sum of repeating decimals as a repeating decimal:
 $0.\overline{4} + 0.\overline{5} + 0.\overline{7}$.
5. Suppose that $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$. What is the value of
$$\sum_{k=1}^{1000} \frac{1}{k(k+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \Lambda + \frac{1}{1000 \cdot 1001} ?$$
6. A bathtub has two faucets. Faucet number one can fill the tub in 15 minutes. Both faucets together can fill the tub in 8 minutes. How fast can faucet number two fill the tub alone? Express your answer as a rational number in lowest terms.
7. Let n and m be positive integers. If the greatest common factor of n and m is 7 and the least common multiple is 1001, what are the values of n and m ?

8. Consider the polynomial $x^3 + 4x^2 - 11x - 30$. Below you will find the following work involving synthetic division.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -11 & -30 \\ & & 3 & 21 & 30 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

Use this to completely factor $x^3 + 4x^2 - 11x - 30$.

9. Consider high school seniors taking math, science, and economics. Ten students take all three courses. Five students take only math and science, while eight students take math and economics only. Three students take only science and economics. If math claims they have 26 students, how many students only take math?
10. Consider the following mathematical expression:

$$\ln y = Kx + \ln b$$

with $K = -2$. and $\ln b = 1$. Find the value of y when $x = 0$.

11. Find *all values* where the following function does not exist:

$$f(x) = \frac{2x}{x^3 - 3x^2 + 2x}$$

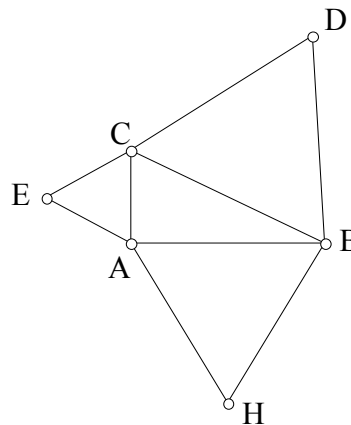
12. Consider the following sequence:

$$1, 1, 2, 3, 5, 8, 13, \dots$$

which is called the *Fibonacci Sequence*. The first number is called F_0 . Find F_{10} .

13. Your local gas mart has been collecting data for daily supply and daily demand. The supply curve is $y = 3x$ and the demand curve is $y = 20 - 2x$. How much gas should be on hand at the start of every day to meet demand?

14. In the picture to the right $\angle CAB$ is a right angle and $\triangle ACE$, $\triangle BCD$, and $\triangle ABH$ are equilateral triangles. If the areas of $\triangle ACE$ and $\triangle ABH$ are 25 and 49 respectively, what is the area of $\triangle BCD$?



15. Two cars start at the same time. The first car heads north at 60 miles per hour (mph). The second car heads east at 80 mph. At what time will the cars be 300 miles apart?

16. Solve for x : $\left(\frac{1}{4}\right)^{2x+8} = (16)^{2x+5}$.

17. A five digit number is called a *valley number* if the first three digits are in descending order and the last three are in ascending order. For example, 74289 and 95368 are valley numbers while 23257, 44369, and 53276 are not valley numbers. How many valley numbers are less than 30000?