

Questions from the Written Competition for the Fall 1998 Pee-Dee Regional High-School Mathematics Tournament

Sponsored by
The Pee Dee Education Center
and
The Department of Mathematics at Francis Marion University

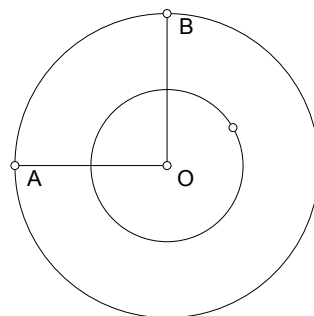
Students had one hour to solve these problems. Space was permitted on the written competition for students to work their problems. Only the questions are written here.



1. A right triangle has legs with lengths $5 - \sqrt{7}$ and $5 + \sqrt{7}$. What is the length of the hypotenuse?

2. Simplify the expression $\frac{(x^2 - 1)(x^4 - 1)(x^6 - 1)}{(x + 1)(x^2 + 1)(x^3 + 1)}$.

3. The circles C_1 and C_2 are concentric with center O . The radius of C_2 is twice that of C_1 and $m\angle AOB = 90^\circ$. If the area of circle C_2 is 40 square units, what is the area of the shaded region?



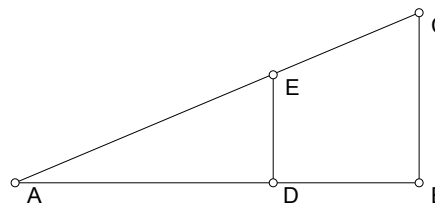
4. Assume that $\sqrt{a} = 0.0117$, what then is $\sqrt{10,000 \cdot a}$?

5. A rational number has a decimal expansion $0.53535353\dots = 0.\overline{53}$. What is the rational number, in lowest terms?

6. If $\log(a) + \log(a^2) + \log(a^3) + \Lambda + \log(a^{10}) = 110$, what is the value of a ? (Note: the logarithms here are base ten.)

7. Suppose $8^x \cdot 4^{(x+3)} = 2^{36}$. What is the value of x ?

8. In the figure to the right ABC and ADE are right angles. If $AB = 35$, $AD = 20$ and $DE = 15$, what will be the value of BC ?



9. The decimal expansion of $\frac{1}{12}$ is $0.0833333\dots$. Its second decimal place is 8 and its fifth decimal

place is 3. What is the 30th decimal place of the decimal expansion of $\frac{1}{37}$?

10. A bathtub has two faucets. Faucet number one can fill the tub in 5 minutes. Faucet number two can fill the tub in 3 minutes. How long will it take the two faucets together to fill the tub. Express your answer as a rational number in lowest terms.

11. If $\cos \theta = 5/13$ and θ is in the fourth quadrant, what is the value of $\tan \theta$? Express your answer as a rational number in lowest terms.
12. The sum of ten numbers is 50. Each number is tripled and then increased by two. What now is the value of the sum of these numbers?
13. In the figure to the right $\triangle ABC$ has an inscribed circle. Further, $m\angle ABC = 90^\circ$, $AB = 8$, and $BC = 6$. What is the radius r of the circle?
14. A car can drive from point A to point B in 8 hours. If the speed is increased by 5 mph it can drive the same distance in 7 hours. What is the distance from point A to point B?
15. If an integer N is divided by 5, 7, or 11, it will have a remainder of 1. What is the smallest positive value of N ?
16. A person's salary is \$10,000. In year 1 he gets a 10% raise. In year 2 he gets a 10% reduction. What is his final salary?
17. If $\log_2 x = 6$, what is the value of $\log_8 x^{10}$?
18. How many integers from 1 to 700 are evenly divisible by both 5 and 7?

