Questions from the Written Competition for the Fall 1997 Pee-Dee Regional High-School Mathematics Tournament

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The Pee Dee Education Center
and
The Department of Mathematics at Francis Marion University

Students had one hour to solve these problems. Space was permitted on the written competition for students to work their problems. Only the questions are written here. Diagrams also appeared on the competition that the students ha; these diagrams do not show up on this document.

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1. What is the rational number (in lowest terms) that is represented by the repeating decimal 0.23232323... ?

2. If $n! = n(n-1)(n-2) \wedge 2 \cdot 1$, then simplify $\dfrac{2! \cdot 3! \cdot 6! \cdot 7! \cdot 10!}{1! \cdot 4! \cdot 5! \cdot 8! \cdot 9!}$.

3. The radius of circle C_2 is three times the radius of circle C_1. If A_1 is the area of C_1 and A_2 is the area of C_2, then what is the value of A_2/A_1 ?

4. Let f be a function defined by the fact that $f(x) = x/2$ if x is an even integer, and by the fact that $f(x) = (x+1)/2$ if x is an odd integer. Evaluate $f(f(f(f(10))))$.

5. Let $\text{int}(x)$ be the integer part of a positive real number x. For example: $\text{int}(1/2) = 0$, $\text{int}(3) = 3$, and $\text{int}(7.5) = 7$. Evaluate the sum

$\text{int}(1/2) + \text{int}(2/2) + \text{int}(3/2) + \text{int}(4/2) + \text{int}(5/2) + \ldots + \text{int}(200/2)$.

6. Let $\overline{EC}$ be parallel to $\overline{AB}$, $CD = 3$, and $BC = 5$. If the area of triangle ABD is 64 square units, then what is the area of triangle CDE ?

7. John can cut 1 acre in 3 hours. Jane can cut 3 acres in 4 hours. How long will it take Jane and John to cut a 13 acre lawn if they work together and cut at the same rate? Express your answer in hours.

8. If $\cos x = 12/13$ and x is in the first quadrant, what is the value of $\tan x$?

9. Let $f(x) = x(x+1)(x+2)(x+3)$. Simplify $x\left[\dfrac{f(x+1) - f(x)}{f(x)}\right]$.

10. If $n! = n(n-1)(n-2) \wedge 2 \cdot 1$, then simplify $\dfrac{11! - 10!}{10!}$.

11. Simplify $\log_2(3) \cdot \log_3(4) \cdot \log_4(5) \cdot \log_5(6) \cdot \log_6(7) \cdot \log_7(8)$.

12. The circle in the diagram has a circumference of 120 and the measure of $\angle AOB$ is 24 degrees. What is the length of the arc AB? [Evidently O is the center of the circle, and A and B are on its perimeter. — Editor]

13. Let $f(x) = ax^2 + bx + c$ be a second degree polynomial and let $x - 3$ be one of its factors. Suppose $f(1) = 12$ and $f(2) = 25$. Find the values of a, b, and c.

14. Write the following difference of two repeating decimals as a repeating decimal:
$$2.1111\wedge - 1.8888\wedge$$

15. Assume a man is able to count from 1 to 10^{15} in 5 years. If he counts at the same rate, how long would it take him to count to 10^{16}?

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