THE · 38TH · PEE – DEE · REGIONAL · HIGH – SCHOOL

MATHEMATICS · TOURNAMENT

Written Competition

SPONSORED · BY · FRANCIS · MARION · UNIVERSITY

MU · ALPHA · THETA · AND · THE · PEE · DEE · EDUCATION · CENTER

TUESDAY · 2014 · DECEMBER · 02

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name *as you wish it to appear if you are given an award.*

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored.

Because the judges must score over 250 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:	**Unacceptable**	**Acceptable**
One must perform all arithmetic that evaluates to an integer.	$2^2 \cdot 3^3 \cdot 5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must "take out" all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$
All ratios must be written as a pure number in conventional notation. Translate ":" as "/" and, if necessary, simplify the resulting fraction.	$1:2$	$\frac{1}{2}$

↑ Name. (Print neatly and fully.)

↑ High School.

* Used only in tie-breaking.

—— For official use only ——

Page **1**. (# 1, 2, 3)	Page **2**. (# 4, 5, 6)
Page **3**. (# 7, 8, 9)	Page **4**. (# 10, 11, 12)
Page **5**. (# 13, 14, 15)	Page **6**. (# 16, 17, 18)
Total Correct	**Weighted Sum***

Awards

1. Simplify completely: $(\sqrt{4}+\sqrt{9})^2$

2. Simplify completely: $\sqrt{3^2+4^2}$

3. Simplify completely: $\log_2 (2^3+2^3)$

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 1:	**Answer to Problem 2:**	**Answer to Problem 3:**

4. Matthew takes three minutes to lay two bricks. When Matthew and Mark work together, they can lay three bricks in two minutes. How long will it take Mark to lay one brick if he works by himself. (Assume Matthew and Mark work independently of one another.) To gain credit for this problem, you must also answer in minutes and seconds.

5. Simplify completely: $\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right)^2\cdot\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)^2$

6. Simplify completely: $\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\right)^3\cdot\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)^3$

— In order to receive credit, answers must appear in these boxes and be expressed in the form specified. —

Answer to Problem 4:.	**Answer to Problem 5:**	**Answer to Problem 6:**
Mark can lay one brick in ______ min + ______ sec		

In the problems below, the *root* of a polynomial f is a real or complex number x so that $f\,(x) = 0$. In the blanks below, write negative numbers for those coefficients that are, indeed, negative. For example, the polynomial $x^2 - 20x + 405$ should be reported as $x^2 + \underline{\ -20\ }\ x + \underline{\ 405\ }$.

7. Once upon a time, there was a linear polynomial. Its leading coefficient was 3. Its single root was 12. What was the linear polynomial?

8. Once upon a time, there was a quadratic polynomial. Its leading coefficient was 2. The sum of its two roots was 3. The product of its two roots was 4. What was the quadratic polynomial?

9. Once upon a time, there was a cubic polynomial. Its leading coefficient was 5. One of its roots was 6. The sum of its two other roots was 7. The product of these two other roots was 8. What was the cubic polynomial?

— In order to receive credit, answers must appear in these boxes. —

Answer to Problem 7:.	Answer to Problem 8:	Answer to Problem 9:
$3\,x + \_\_\_\_\_$	$2x^2 + \_\_\_\_\_\ x + \_\_\_\_\_$	$5x^3 + \_\_\_\_\_\ x^2 + \_\_\_\_\_\ x + \_\_\_\_\_$

Consistently with the protocols given on the front of this competition, report all ratios as a normal-looking fraction properly simplified. That is, report answers that look like "$\frac{3}{4}$" or "3 / 4", not "3 : 4", nor "3 to 4", and certainly not "$\frac{6}{8}$".

10. A circle is inscribed in a square, which is inscribed in a circle. What is the ratio of the area of the smaller circle to the area of the larger circle?

11. A sphere is inscribed in a cube, which is inscribed in a sphere. What is the ratio of the volume of the smaller sphere to the volume of the larger sphere?

12. A two-by-one rectangle is by definition a rectangle that is twice as long as it is wide. An ellipse is inscribed in a two-by-one rectangle, which is inscribed in a circle. What is the ratio of the area of the ellipse to the area of the circle?

Be careful that no answer appear with a radical in the denominator. Rationalize denominators as needed.

Answer to Problem 10:	**Answer to Problem 11:**	**Answer to Problem 12:**

13. Three circles, each of radius 1 unit, are drawn tangent to one another, as shown. What is the area of the small region (shaded in the diagram) surrounded by the three circles?

14. Three circles, each of radius 1 unit, are drawn tangent to one another, as shown. What is the radius of the small circle that is tangent to all three circles?

15. Two circles, each of 1 radius, and one circle of radius *x* units, are drawn tangent to one another, as shown. What is the distance from one point of tangency to the other through the larger circle? Answer as a function of *x*.

Figure for Problem 13

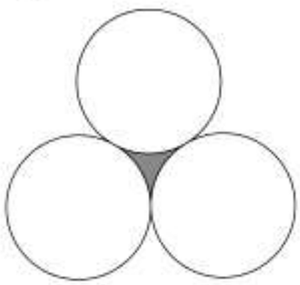


Figure for Problem 14

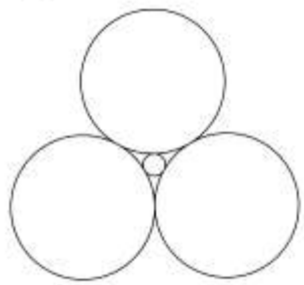
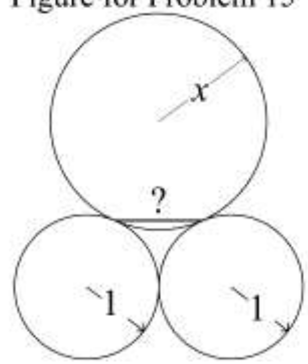


Figure for Problem 15



Be careful that no answer appear with a radical in the denominator. Rationalize denominators as needed.

Answer to Problem 13:	**Answer to Problem 14:**	**Answer to Problem 15:**
square units	units	? = units

In the problems that follow, the units in the answer are given as rotations, where

$$1 \text{ rotation } = 2\pi \text{(radians)} = 360°.$$

You must answer in rotations, not degrees or radians.

16. A fixed larger circle, of radius four units, has a smaller circle, of radius one unit, outside of it. How many rotations does the smaller circle undergo as it rolls once around the larger circle without slipping?

17. A fixed larger circle, of radius four units, has a smaller circle, of radius one unit, inside of it. How many rotations does the smaller circle undergo as it rolls once around the larger circle without slipping?

18. A fixed smaller circle, of radius one unit, is surrounded by a larger circle, of radius four units. How many (perhaps fractional) rotations does the larger circle undergo as it rolls once around the smaller circle without slipping?

Figure for Problem 16

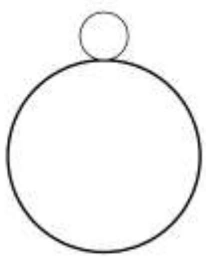


Figure for Problem 17

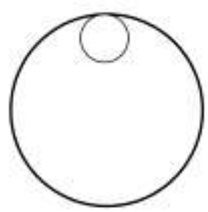
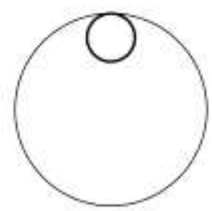


Figure for Problem 18



— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 16:	Answer to Problem 17:	Answer to Problem 18:
rotations	rotations	rotations

Answer to Ultimate Tiebreaker: