THE · 36TH · PEE – DEE · REGIONAL · HIGH – SCHOOL
MATHEMATICS · TOURNAMENT

Written Competition

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AND · THE · PEE · DEE · EDUCATION · CENTER
TUESDAY · 2012 · DECEMBER · 04

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name *as you wish it to appear if you are given an award.*

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored.

Because the judges must score over 250 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:

	Unacceptable	Acceptable
One must perform all arithmetic that evaluates to an integer.	$2^2 \cdot 3^3 \cdot 5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must "take out" all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$
All ratios must be written as a pure number in conventional notation. Translate ":" as "/" and, if necessary, simplify the resulting fraction.	1 : 2	$\frac{1}{2}$

↑ Name. (Print neatly and fully.)

↑ High School.

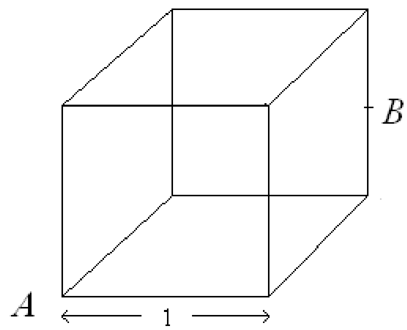
* Used only in tie-breaking.

—— For official use only ——

Page **1**. (# 1, 2, 3)	Page **2**. (# 4, 5, 6)
Page **3**. (# 7, 8, 9)	Page **4**. (# 10, 11, 12)
Page **5**. (# 13, 14, 15)	Page **6**. (# 16, 17, 18)
Total Correct	**Weighted Sum***

Awards

1. A cube with a side of 1 unit is shown at right. The point A is at one vertex of the cube, but the point B is at the midpoint of one of the edges furthest from A. What is the length of the shortest path from point A to point B where the path is constrained to lie along the edges of the cube?



2. The continued fraction shown at right provides an approximation to $\sqrt{2}$. Simplify the continued fraction until the result appears plainly as an integer over an integer or as a mixed fraction with largest possible whole part.

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}$$

3. Simplify thoroughly using various trigonometric identities: $6\cos^2(x) - 3\cos(2x)$.

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 1:	**Answer to Problem 2:**	**Answer to Problem 3:**
units		

4. Simplify completely: $\left(x+\dfrac{1}{x}\right)^2-\left(x^2+\dfrac{1}{x^2}\right)$

5. Solve for x and answer exactly: $x+\dfrac{1}{x}=5$

6. A number (call it x) and its reciprocal sum to 5: $x+\dfrac{1}{x}=5$

What do the cube of this number and *its* reciprocal sum to: $x^3+\dfrac{1}{x^3}\ =\ ?$

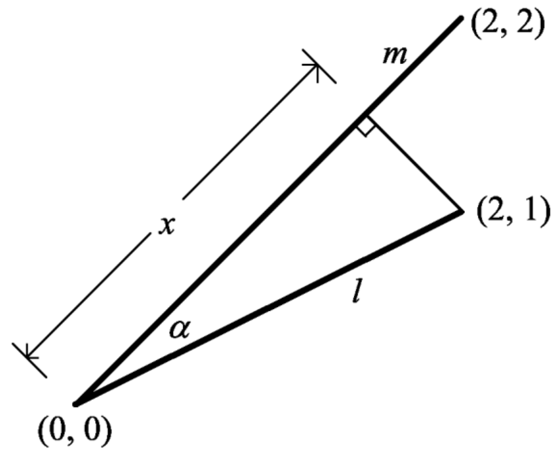
— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 4:	**Answer to Problem 5:**	**Answer to Problem 6:**
	$x\ =$	$x^3+\dfrac{1}{x^3}\ =$

7. Mary has a loop of string exactly 24 units long. In how many ways can Mary use the string to create distinct, non-degenerate triangles with integer sides and perimeter of 24 units? *Note:* Distinct triangles are not congruent to one another, and non-degenerate ones have non-zero area.

8. Line l joins the points (0,0) and (2,1), and line m joins (0, 0) to (2, 2). What is the tangent of the angle between lines l and m?

9. A perpendicular is dropped from (2, 1) to the line m of Problem 8. How far does the foot of this perpendicular fall from the origin?



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Answer to Problem 7:	Answer to Problem 8:	Answer to Problem 9:
	$\tan\alpha =$	$x =$

10. Jupiter and Saturn are on a circular racetrack. (The celestial mechanics in this problem are vastly oversimplified!). At time zero, Jupiter and Saturn are on the same point on the racetrack. Jupiter can complete a circuit around the track in 12 years, but Saturn takes 30 years to complete a full circuit. In how many years will Jupiter and Saturn be at the same point as each other (not necessarily the starting point) along this racetrack?

11. In how many years will the simplified planets Jupiter and Saturn of Problem 11 both be together *and* at the original starting point?

12. In a certain class of 100 students, some passed the last examination and some failed. Those who passed had an average of 90 on it, but those who failed had an average of 50. The class average on the test (passers and failers combined) was 82. How many students passed the examination?

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 10:	**Answer to Problem 11:**	**Answer to Problem 12:**
years	years	students

The table at right of various integral powers of 2 may be of assistance in answering some or all of the questions on this page.

13. Evaluate 5^8.

14. Evaluate $(1 + i)^{17}$, where $i = \sqrt{-1}$. Answer in the form $a + bi$, where a and b are real numbers.

15. It is well known that $\pi \approx 3.14159265$ when written base ten with eight decimal digits beyond the decimal point. What is the number π when written base *two* with eight *binary* digits beyond the decimal point? *Make sure that all eight binary digits appear beyond the decimal point.*

2^{-10}	=	0.0009765625
2^{-9}	=	0.001953125
2^{-8}	=	0.00390625
2^{-7}	=	0.0078125
2^{-6}	=	0.015625
2^{-5}	=	0.03125
2^{-4}	=	0.0625
2^{-3}	=	0.125
2^{-2}	=	0.25
2^{-1}	=	0.5
2^{0}	=	1
2^{1}	=	2
2^{2}	=	4
2^{3}	=	8
2^{4}	=	16
2^{5}	=	32
2^{6}	=	64
2^{7}	=	128
2^{8}	=	256
2^{9}	=	512
2^{10}	=	1024

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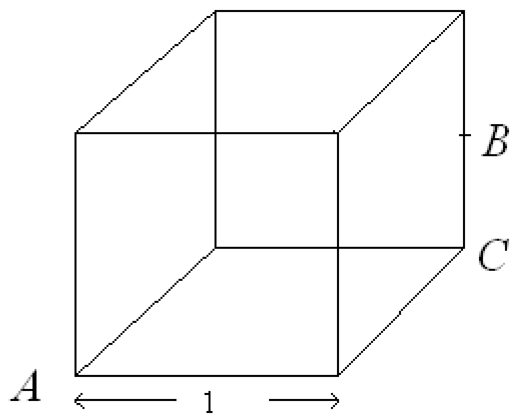
Answer to Problem 13:	**Answer to Problem 14:**	**Answer to Problem 15:**
		$\pi \approx$ base two

A cube with a side of 1 unit is shown at right. The point A is at one vertex of the cube, but the point B is at the midpoint of one of the edges furthest from A. The cube has twelve *edges*, which are line segments, and six *faces*, which are squares.



16. What is the length of the shortest path from A to B, as the path is permitted to travel through space?

17. What is the length of the shortest path from point A to point B where the path is constrained to lie on the faces of the cube?

18. The cube shown is carefully placed in a tank of water. Points A and B are on the surface of the water, but the cube is rotated so that the point C be as low in the water as possible. What is the volume of the water displaced by the cube when the cube is partly submerged in this fashion?

Note well: *Remember throughout to perform all simplifications indicated on the front of this competition and to have answers in the proper boxes. The judges are instructed to reject answers that do not adhere to these protocols even if they are otherwise right.*

Answer to Problem 16:	**Answer to Problem 17:**	**Answer to Problem 18:**
units	units	cubic units

Answer to Ultimate Tiebreaker: