THE · 33RD · PEE – DEE · REGIONAL · HIGH – SCHOOL

MATHEMATICS · TOURNAMENT

Written Competition

SPONSORED · BY · FRANCIS · MARION · UNIVERSITY

AND · THE · PEE · DEE · EDUCATION · CENTER

TUESDAY · 2009 · DECEMBER · 08

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name *as you wish it to appear if you are given an award.*

During the competition, no calculators are allowed. Cellphones are also strictly forbidden.

Each final answer must be placed in its proper answer box or it will not be scored.

Because the judges must score over 300 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:

	Unacceptable	Acceptable
One must perform all arithmetic that evaluates to an integer.	$2^2\,3^3\,5$	540
One must cancel all common factors in fractions of two integers.	4/6	2/3
In writing fractions, one must choose *either* an integer over an integer, *or* a mixed fraction with largest possible whole part.	$2+\frac{5}{3}$	$\frac{11}{3}$ or $3+\frac{2}{3}$
In writing square-roots, one must "take out" all perfect squares.	$\sqrt{24}$	$2\sqrt{6}$
One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\sqrt{2}$
	$\frac{2}{\sqrt{7}-1}$	$\frac{\sqrt{7}+1}{3}$
All ratios must be written as a pure number in conventional notation. Translate ":" as "/" and, if necessary, simplify the resulting fraction.	1 : 2	$\frac{1}{2}$

↑ Name. (Print neatly and fully.)

↑ High School.

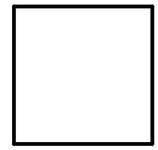
—— *For official use only* ——

Page **1**. (# 1, 2, 3)	Page **2**. (# 4, 5, 6)
Page **3**. (# 7, 8, 9)	Page **4**. (# 10, 11, 12)
Page **5**. (# 13, 14, 15)	Page **6**. (# 16, 17, 18)
Total Correct	**Weighted Sum***

* Used only in tie-breaking.

Awards

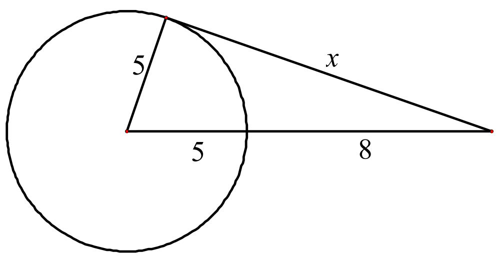
1. A square has an area of 3 square units. What is its perimeter?



2. Simplify the following rational approximation for the golden ratio until the result is an integer over an integer and with all common factors canceled.

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}$$

3. How far is a point that is 8 units away from a circle of radius 5 from its point of tangency with that circle?



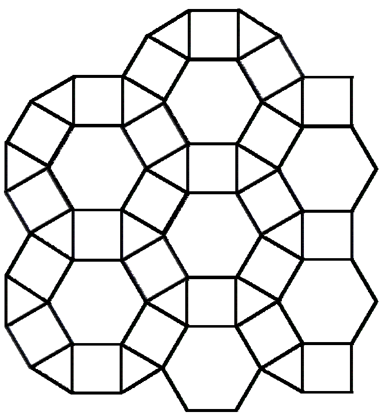
— Answers to problems <u>must</u> appear in these boxes in order to receive credit. —

Answer to Problem 1:	**Answer to Problem 2:**	**Answer to Problem 3:**
		$x =$

4. Bonny and Clyde were driving away from the scene of their last crime in separate cars, each at a uniform rate though perhaps committing felony reckless endangerment as they drove. Clyde was driving twice as fast as Bonny. After 3 hours, Clyde went 165 miles further than did Bonny. How fast was Clyde driving? *Answer with number and units. Answers without proper units will be marked wrong.*

5. Solve for x and simplify your answer completely: $x^2 + 2\sqrt{2}\,x = 2$.
(Lest there be any doubt, the second x is *outside* the radical.)

6. A portion of an Archimedian tessellation is shown below. It consists entirely of equilateral triangles, squares, and regular hexagons. If the tessellation were extended to infinity, what fraction of the tiles would be triangles, what fraction would be squares, and what fraction would be hexagons?



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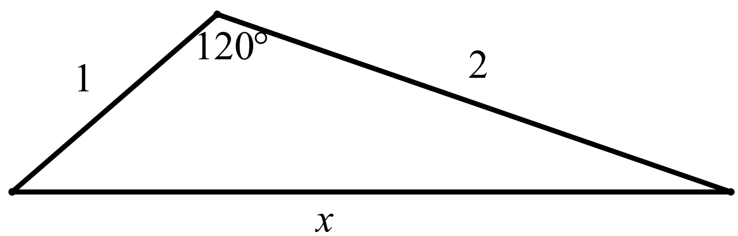
Answer to Problem 4:	Answer to Problem 5:	Answer to Problem 6:		
	$x^2 + 2\sqrt{2}\,x = 2$ if, and only if, $x =$ or $x =$	Fraction of tiles that would be triangles	Fraction of tiles that would be squares	Fraction of tiles that would be hexagons

7. Mary has three dice, each in the shape of a cube with the numerals "1" through "6" on the sides. What is the probability that Mary, upon rolling these dice, can form the number 554 with the numbers on their tops? *To gain credit for this problem, you must also reduce the fraction to lowest terms.*

8. Evaluate $(1 + i)^{16}$, where $i = \sqrt{-1}$.

9. Solve for x:



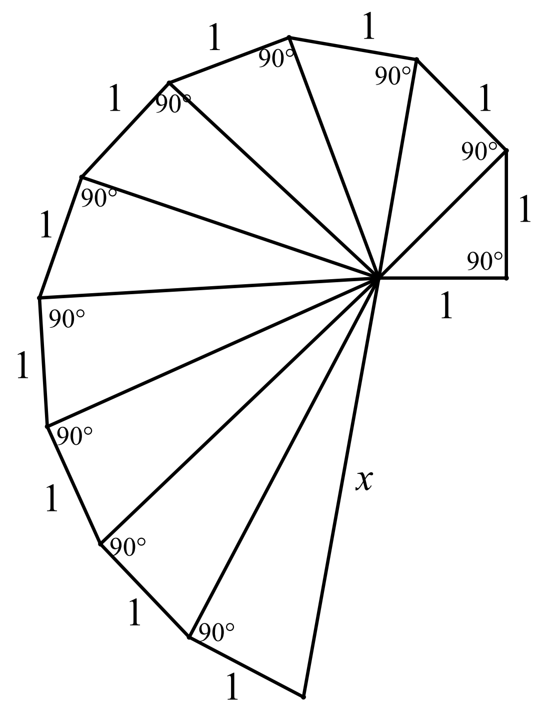
— Answers to problems must appear in these boxes in order to receive credit. —

Answer to Problem 7:	Answer to Problem 8:	Answer to Problem 9:
		$x =$

10. The quadratic polynomial $x^2 + bx + c$ evaluates to 3 when $x = 2$ and when $x = 5$. Find the parameters b and c from this information.

11. In what base B is 121_B equal to forty-nine base ten?

12. Solve for x:



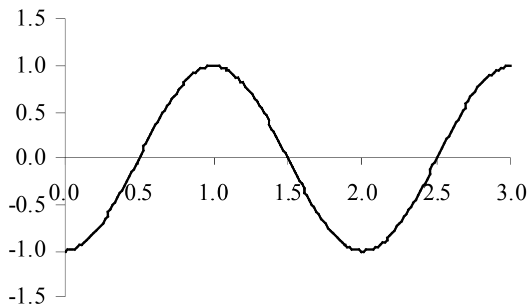
— Answers to problems must appear in these boxes in order to receive credit. —

Answer to Problem 10:	Answer to Problem 11:	Answer to Problem 12:
$b =$ $c =$	$B =$	$x =$

13. Evaluate 3^{215} mod 10. That is, if 3^{215} were computed, what would be the units digit of the final result?

14. Lenny Oiler has secured a job counting attendees at the entrance of the National Mathematics Convention. He grabs a mechanical counter and sets out for work. At the end of the day, the counter shows the number 5695. But Mr. Oiler discovers that the counter is defective: whenever the counter clicks from any digit of 7, it skips 8 and goes directly to 9. How many people actually attended the conference?

15. Below is the graph of a sinewave, $\{y = \sin(\omega t + \varphi)\}$, where t is on the horizontal axis, and y is on the vertical axis, and where ω (read "omega") and φ (read "phi") are parameters (that is, ω and φ are fixed real numbers). The amplitude of this sinewave is 1, but horizontally the sinewave has been stretched and shifted and in fact its first three crossings on the t-axis are at 1/2, at 3/2, and at 5/2. Find the parameters ω and φ from this information. *The input to the sine function is understood to be in radians, not degrees. Multiple right answers are in fact possible. Answer any <u>one</u> of them.*



— Answers to problems <u>must</u> appear in these boxes in order to receive credit. —

Answer to Problem 13:	Answer to Problem 14:	Answer to Problem 15:
		$\omega =$ $\varphi =$

16. A room has a corner where both walls and the floor meet at right angles with one another. A sphere of radius 1 unit is placed so that it is tangent to both walls and is tangent also to the floor. What is the distance from the corner of the room to the center of the sphere?

17. A smaller sphere is set in the space between the sphere in Problem 16 and the corner of the room. This smaller sphere is tangent to both walls, the floor, and the sphere of radius 1 unit. What is the radius of this smaller sphere? *Be sure that your answer is properly simplified.*

18. A larger sphere is now placed *outside* the sphere of radius 1. It too is tangent to both of the walls, the floor, and the sphere of radius 1, but is *larger* than the sphere of radius 1. What is the ratio of the radius of the smaller sphere described in problem 17 to the radius of the large sphere described here in Problem 18? *Answer as a pure number properly simplified.*

Note well: *Remember throughout to perform all simplifications indicated on the front of this competition and to have answers in the proper boxes. The judges are instructed to reject answers that do not adhere to these protocols even if they are otherwise right.*

Answer to Problem 16:	**Answer to Problem 17:**	**Answer to Problem 18:**

Answer to Tiebreaking Question: