THE · 40TH · PEE – DEE · REGIONAL · HIGH – SCHOOL
MATHEMATICS · TOURNAMENT
Written Competition
SPONSORED · BY · FRANCIS · MARION · UNIVERSITY
MU · ALPHA · THETA · AND · THE · PEE · DEE · EDUCATION · CENTER
TUESDAY · 2016 · DECEMBER · 06

Instructions

Do not turn over this page until instructed to do so.

Neatly print (not sign) your name in the space below
as you wish it to appear if you are given an award.

During the competition, no calculators are allowed. Cellphones also are strictly prohibited.

Each final answer must be placed in its proper answer box or it will not be scored.

Because the judges must score over 350 papers in under an hour, they have not time to deal with unsimplified answers. Therefore:

One must perform all arithmetic that evaluates to an integer.

One must cancel all common factors in fractions of two integers.

In writing fractions, one must choose *either* an integer over an integer *or* a mixed fraction with largest possible whole part.

In writing square-roots, one must "take out" all perfect squares.

One must rationalize the denominator whenever a square-root appears in the bottom of a fraction. After rationalization, one must also be sure to cancel any common factors.

Unacceptable	Acceptable
$2^2 \cdot 3^3 \cdot 5$	540
4/6	2/3
$2 + \dfrac{5}{3}$	$\dfrac{11}{3}$ or $3 + \dfrac{2}{3}$
$\sqrt{24}$	$2\sqrt{6}$
$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{2}}{2}$ or $\dfrac{1}{2}\sqrt{2}$
$\dfrac{2}{\sqrt{7}-1}$	$\dfrac{\sqrt{7}+1}{3}$

If a problem specifies that an answer is to be rounded to the nearest whole number, then you *must* round in order to receive credit.

—— *For official use only* ——

↑ Name. (Print neatly and fully.)

↑ High School.   * Used only in tie-breaking.

Awards

Page **1**. (# 1, 2, 3)	Page **2**. (# 4, 5, 6)
Page **3**. (# 7, 8, 9)	Page **4**. (# 10 to 15)
Page **5**. (# 16, 17, 18)	Page **6**. (19, 20, 21)
Total Correct	**Weighted Sum***

The integers are elements from the set $\{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$. A *rectangular integral garden* is a rectangular plot of land whose length and width are both integers, when measured in feet. These gardens must be rectangles; no other shape is allowed. Below are shown three integral gardens, with 1, 2, and 3 square feet of garden, respectively, and they are *completely surrounded* by white tiles, each tile being 1 ft × 1 ft. An integral garden of 1 square foot can be most efficiently surrounded by 8 white tiles, as shown below. Two other integral gardens, completely surrounded by white tiles, are also shown.

1. How many white tiles are needed completely to surround a rectangular integral garden of area 2 square feet? *Don't let the easy ones stump you.*

2. How many white tiles are needed completely to surround a rectangular integral garden of area 3 square feet?

3. Dr. Pangloss has created a rectangular integral garden with 7 square feet of area. How many white tiles are needed to surround it completely, assuming a most efficient design?
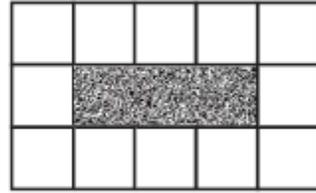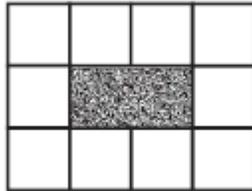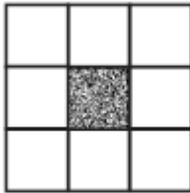
— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 1:	Answer to Problem 2:	Answer to Problem 3:
white tiles	white tiles	white tiles

4. Martin has created a rectangular integral garden whose area is 9 square feet. How many white tiles are needed to surround it completely, assuming a most efficient design?

5. How many white tiles are needed completely to surround Cunégonde's rectangular integral garden of 45 square feet, assuming a most efficient design?

6. Candide has 98 white tiles. What is the area of the largest rectangular integral garden that Candide can surround completely with his white tiles?

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 4:	Answer to Problem 5:	Answer to Problem 6:
white tiles	white tiles	square feet

7. "My net worth is 25% larger than your net worth," said Stud-Man to Hot-Rod. "That may be true," said Hot-Rod, "but my net worth is not 25% less than yours." How much less is Hot-Rod's net worth than Stud-Man's, rounded expertly, if needed, to the nearest whole percent?

8. Daisy-Belle is a young lady who is well aware of her worth, or, at least, is well aware of her *net* worth. "If I marry Stud-Man," she said, "our combined net worth will be only one percent larger than if I combine my net worth with Hot-Rod's." How many *times* larger is Daisy-Belle's net worth than Hot-Rod's, rounded expertly, if needed, to the nearest whole number?

9. Jay Gatsby could not help overhearing this discussion and joining in. "My net worth is seven hundred thousand dollars and is four times as much as all three of your net worths put together," he boasted. What is Stud-Man's net worth, rounded expertly, if needed, to the nearest whole dollar?

— In order to receive credit, answers must appear in these boxes and be expressed in the form specified. —

Answer to Problem 7:.	Answer to Problem 8:	Answer to Problem 9:
__ __ %		dollars

A pentagram is inscribed in a regular pentagon *ABCDE*. Each side of the pentagon is of length 1, and the entire figure has a perfect, five-fold rotational symmetry in addition to a bilateral symmetry at each of its vertices. Consequently, all the angles marked α are indeed equal to each other, and all the angles marked β are also equal to one another. A portion of the lower part of the diagram is extracted for further questioning.

10. What is the angle α in the diagram? *You must answer in degrees.*

11. What is the angle β in the diagram? *You must answer in degrees.*

12. What is the angle γ in the diagram? *You must answer in degrees.*

13. What is the length of x in the diagram?

14. No more Mr. Nice Guy. What is the length of y in the diagram? The length of y is uniquely determined from the data. Multiple answers, such as those with "$\pm$" in them, cannot be right.

15. Find an exact formulation for $\sin 18°$. *Your answer will involve square-roots, which must be simplified according to the instructions on the front page.*
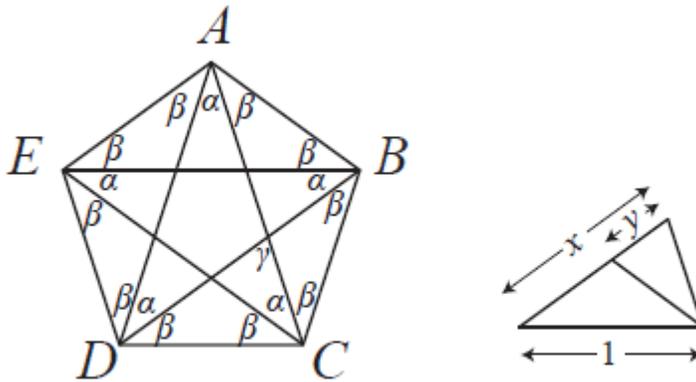
— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 10:	Answer to Problem 11:	Answer to Problem 12:	Answer to Problem 13:	Answer to Problem 14:	Answer to Problem 15:
degrees	degrees	degrees	units	units	

A googol is the number that results when the Arabic number 1 is followed by an hundred 0's. In simpler terms, a googol $= 10^{100}$, and in base ten is written as shown below.

16. The U.S. Federal Debt, in dollars, is approximately twenty trillion: 20,000,000,000,000.

How many times larger is a googol than the U.S. Federal Debt in dollars:

$$\frac{\text{a googol}}{\text{U.S. Federal Debt in dollars}} \; = \; ?$$

To receive credit for this problem, you must answer to three significant digits of accuracy and in scientific notation, for example 1.23×10^{45}. The form in the answer box should help you out: put an Arabic numeral in each underlined spot.

17. What is the remainder when a googol is divided by 8?

18. What is the remainder when a googol is divided by 9?

10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000

— In order to receive credit, answers must appear in these boxes and be expressed in the form specified. —

Answer to Problem 16:.	Answer to Problem 17:	Answer to Problem 18:
__ . __ __ $\times\ 10^{\,-\,-}$		

19. What is the remainder when a googol is divided by 72? (Note: $72 = 8 \times 9$.)

20. What is the remainder when a googol is divided by 7?

21. What is the remainder when a googol is divided by 504? (Note: $504 = 7 \times 8 \times 9$.)

— In order to receive credit, answers must appear in these boxes and be properly simplified. —

Answer to Problem 19:	Answer to Problem 20:	Answer to Problem 21: