

A Touching Problem: P.o.W. Oct. 24, 2016

Assume x, y, z have radius 1. The triangle that connects the centers of x, y, z is equilateral with side length 2. The medians of the triangle (lines from vertex to midpoint of opposite side) meet at a centroid that is both the center of circle A and the center of circle B . For equilateral triangles, the centroid divides the median in a 2:1 ratio from vertex to centroid to midpoint to centroid. The median is $\sqrt{3}$ in length by Pythagorean theorem, so from the vertex to centroid, the median has length $\frac{2\sqrt{3}}{3}$. The radius of circle A is $\frac{2\sqrt{3}}{3} - 1 = \frac{2\sqrt{3} + 3}{3}$ and of circle B is $\frac{2\sqrt{3}}{3} + 1 = \frac{2\sqrt{3} + 3}{3}$. Thus, their ratio is

$$\frac{\frac{2\sqrt{3} + 3}{3}}{\frac{2\sqrt{3} - 3}{3}} = \frac{2\sqrt{3} + 3}{2\sqrt{3} - 3} = 7 + 4\sqrt{3}$$

