PoW Solution 11/14/16

Let N be the number of integers written on the board. The numbers to be replaced have form $a = md$ and $b = nd$ where m, n are relatively prime, $d = \gcd(a, b)$, $n, m, d > 0$.

Then $\operatorname{lcd}(a, b) = mnd$. Thus a & b are replaced by d and mnd.

<u>notice</u>: $P = a \cdot b = md \cdot nd = d \cdot mnd$, so the product does not change.
(ie only finitely many ways to write P as a product of N integers)

<u>notice</u> $S = a + b = nd + md = d(n+m)$
and $d + mnd = d(1 + mn)$

so $d(1+mn) - d(n+m) = d(m-1)(n-1) \geq 0$.

So sum is nondecreasing. Thus once we leave a state we cannot return to it. If m and/or n are 1, the sum no longer changes. Thus, we will end up in one state.